Cause-deleted life tables. Application to Lithuania, 2011

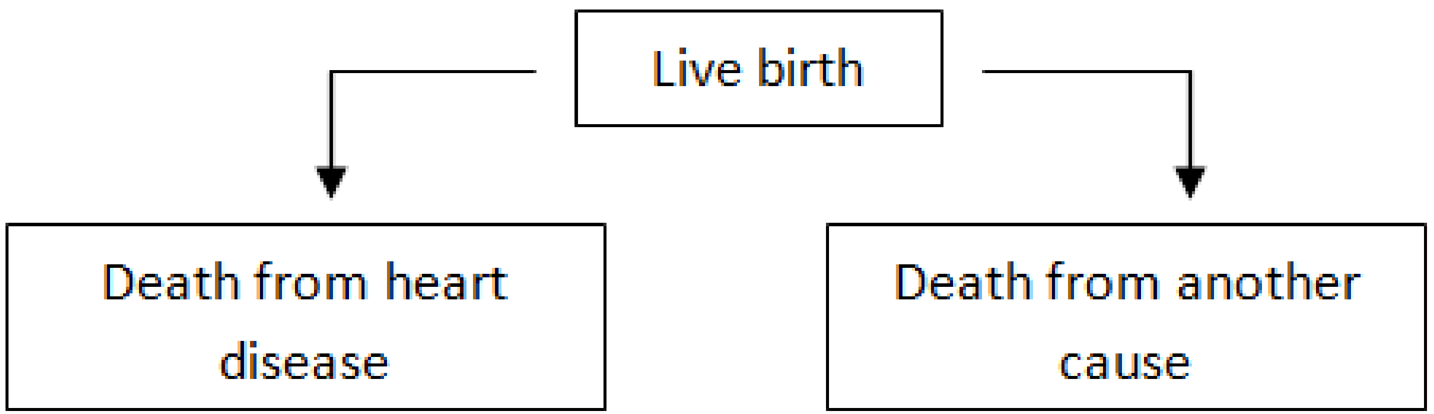
Markéta Pechholdová
INED / University of Economics in Prague

Lithuanian Social Research Centre
Teaching seminar „Causes of death: data and methods of analysis“
February 23-24, 2015 Vilnius
pechholdova@gmail.com

The research is funded by EU structural assistance (European Social Fund) to Lithuania under the measure VP-1-3.1-ŠMM-07-K "Support to Research Activities of Scientists and Other Researchers (Global Grant)" project Nr. VP-1-3.1-ŠMM-07-K-02-067

How many ways to die?

- Classic life table - two states (alive X dead)
  - one mode of exit (death)
- However…



Multiple decrement process

- More than one mode of exit
- More common in demography than single decrement processes
- Any suggestions about multiple decrement processes?
- Migration: move to different places, Marriage: divorce and widowhood
- Competing risks

The theory: example of a watch

- A watch can operate only if all parts are functioning
- Each part has its own life table
- The probability that a watch will work to time (age) x is the product of the independent* probabilities that each component will survive to that age

$$p_x = p_x^{*1} \cdot p_x^{*2} \cdot p_x^{*2} \cdots\cdots p_x^{*k}$$

Cause-deleted life tables

- What a life table would look like if only cause i would operate to diminish the cohort?
- If we are interested in cause –i (=all except i), we produce a „cause-deleted table“
- Cause-deleted life tables provide a clear measure of the public health significance of a particular disease or injury process

Model assumptions

Life table model assumptions (general):

1. No change: death rates are constant across time
2. No migration: birth is the only entrance, death is the only exit
3. No heterogeneity: every individual has the same probability of dying at age *x* (from cause *i)*

Additional assumptions:

1. Each death is due to one cause only (UCD)
2. Probability of dying from any cause is independent from the probability of dying from any other cause
3. Cause-specific death rate is proportional to total death rate:

$$m_x^j = r_x^j \cdot m_x$$

Computational steps

1. Denote k all causes except j

2. Calculate r_x^k: $r_x^k = \frac{m_x^k}{m_x}$

3. Calculate p_x^k:

$$p_x^k = e^{-n \cdot m_x^k} = e^{-n \cdot r_x^k \cdot m_x} = (p_x)^{r_x^k}$$

4. Construct life table

$$p_x^k \to q_x^k \to l_x^k \to L_x^k \to T_x^k \to e_x^k$$

Life table: notation

${}_nm_x$ = death rate of the life table cohort
${}_na_x$ = number of life years lived by those who die
${}_nq_x$ = probability of dying
${}_np_x$ = probability of surviving
${}_nd_x$ = death counts of the life table cohort
l_x = number of survivors
${}_nL_x$ = person-years lived in the age interval
T_x = total person-years lived above age x
e_x = remaining life expectancy

n…x = the age interval x to x+n (n=length of age interval)
x = at exact age x

Life table: equations

${}_n a_x$ – from Human mortality database

l_0 = 100 000, $l_{x+n} = l_x \cdot {}_n p_x$

$${}_n d_x = l_x - l_{x+n}$$

$${}_n L_x = n \cdot l_{x+n} + {}_n a_x \cdot {}_n d_x$$

$$T_x = \sum_{a=x}^{\infty} {}_n L_a$$

$$e_x^o = \frac{T_x}{l_x}$$

Box 3.1 *Period Life Table Construction*

A. Observed data:

$_nN_x$ = *mid-year population in age interval x to x + n*

$_nD_x$ = *deaths between ages x and x + n during the year*

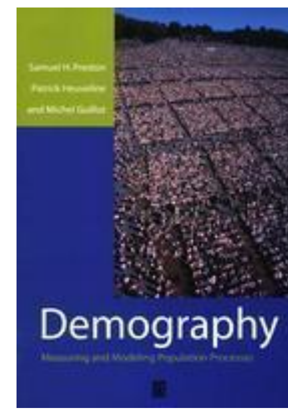
B. Steps for period life table construction:

1. $_nm_x \simeq {}_nM_x = \frac{_nD_x}{_nN_x}$
2. $_na_x$:
   calculated from Coale and Demeny equations shown in table 3.3 under age 5, borrowed from Keyfitz and Flieger above age 5
3. $_nq_x = \frac{n \cdot {}_nm_x}{1 + (n - {}_na_x) \cdot {}_nm_x}$
   $_\infty q_{85} = 1.00$
4. $_np_x = 1 - {}_nq_x$
5. $l_0 = 100,000$
   $l_{x+n} = l_x \cdot {}_np_x$
6. $_nd_x = l_x - l_{x+n}$
7. $_nL_x = n \cdot l_{x+n} + {}_na_x \cdot {}_nd_x$
   (*open-ended interval:* $_\infty L_x = \frac{l_x}{_\infty m_x}$)
8. $T_x = \sum_{a=x}^{\infty} {}_nL_a$
9. $e_x^o = \frac{T_x}{l_x}$

Example: Austria, males, 1992

Age x	$_nN_x$	$_nD_x$	$_nm_x$	$_na_x$	$_nq_x$	$_np_x$	l_x	$_nd_x$	$_nL_x$	T_x	e_x^o
0	47,925	419	0.008743	0.068	0.008672	0.991328	100,000	867	99,192	7,288,901	72.889
1	189,127	70	0.000370	1.626	0.001479	0.998521	99,133	147	396,183	7,189,709	72.526
5	234,793	36	0.000153	2.500	0.000766	0.999234	98,986	76	494,741	6,793,526	68.631
10	238,790	46	0.000193	3.143	0.000963	0.999037	98,910	95	494,375	6,298,785	63.682
15	254,996	249	0.000976	2.724	0.004872	0.995128	98,815	481	492,980	5,804,410	58.740
20	326,831	420	0.001285	2.520	0.006405	0.993595	98,334	630	490,106	5,311,431	54.014
25	355,086	403	0.001135	2.481	0.005659	0.994341	97,704	553	487,127	4,821,324	49.346
30	324,222	441	0.001360	2.601	0.006779	0.993221	97,151	659	484,175	4,334,198	44.613
35	269,963	508	0.001882	2.701	0.009368	0.990632	96,492	904	480,384	3,850,023	39.900
40	261,971	769	0.002935	2.663	0.014577	0.985423	95,588	1,393	474,686	3,369,639	35.252
45	238,011	1,154	0.004849	2.698	0.023975	0.976025	94,195	2,258	465,777	2,894,953	30.734
50	261,612	1,866	0.007133	2.676	0.035082	0.964918	91,937	3,225	452,188	2,429,176	26.422
55	181,385	2,043	0.011263	2.645	0.054861	0.945139	88,711	4,867	432,096	1,976,988	22.286
60	187,962	3,496	0.018600	2.624	0.089062	0.910938	83,845	7,467	401,480	1,544,893	18.426
65	153,832	4,366	0.028382	2.619	0.132925	0.867075	76,377	10,152	357,713	1,143,412	14.971
70	105,169	4,337	0.041238	2.593	0.187573	0.812427	66,225	12,422	301,224	785,699	11.864
75	73,694	5,279	0.071634	2.518	0.304102	0.695898	53,803	16,362	228,404	484,475	9.005
80	57,512	6,460	0.112324	2.423	0.435548	0.564452	37,441	16,307	145,182	256,070	6.839
85	32,248	6,146	0.190585	5.247	1.000000	0.000000	21,134	21,134	110,889	110,889	5.247

Data source: United Nations, 1994.



Source: Preston et al., 2001, p.49

The research is funded by EU structural assistance (European Social Fund) to Lithuania under the measure VP-1-3.1-ŠMM-07-K “Support to Research Activities of Scientists and Other Researchers (Global Grant)” project Nr. VP-1-3.1-ŠMM-07-K-02-067

SCIENCE · ECONOMY · COHESION

EUROPEAN UNION

Creating the Future of Lithuania